Oil Production in Alaska

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INTRODUCTION

We ask several questions about oil extraction from seven production units on Alaska's North Slope.

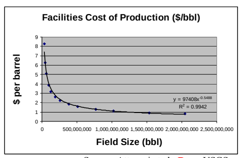
First, can an empirical analysis of extraction decisions enlighten us about the dynamic optimality of producer behavior? To answer this question, we evaluate the limits to complexity in dynamic modeling (i.e., what features are needed to bring theory closer to reality) and the discrepancy between modeled optimal production paths and actual production histories for each of the seven production units.

Second, we evaluate the impact of alternative tax policies on the modeled optimal production paths, with implications for designing a policy to maximize the sum of producer profit and tax revenue.

Implications include the following: 1) for oil producers, evaluation of the dynamic optimality of past production to better inform future decisions; 2) for policy makers, insight into policy tools to encourage more rapid or gradual energy production of oil and other energy resources, to change the allocation of producer profit and tax revenue, and to maximize the net social benefit of profits plus tax revenue.

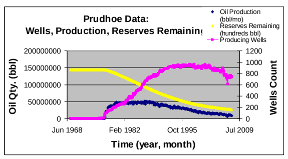
Building Cost Functions

Facilities Cost of Production

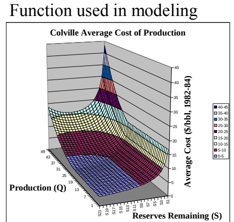


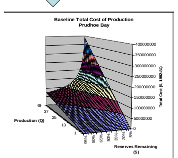
Source: Attanasi et al., Date, USGS

Production and Wells Data

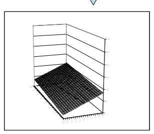


Final Result: Composite Cost Function used in modeling

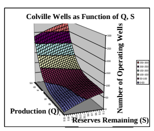




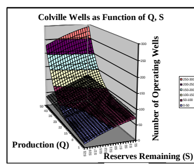
Average Cost ($/bbl)
as a function of production (Q) and reserves remaining (S)



Define a plane
(constant returns to scale)



Define a convex surface
(decreasing returns to scale)



Result: Scalar based on wells

We assume maximization of the discounted stream of future profits as the producers' objective function. Consequently, a function to define the cost of oil extraction is necessary. We estimate the cost function from available data by scaling an average cost function from USGS data by a wells scalar constructed from production data. The result is a production cost surface with marginal cost increasing as reserves are depleted and as production rate exceeds limits to reservoir flow rates.

Analytic vs. Numeric Modeling: the limits to complexity

We found the ordinary differential boundary value problem corresponding to the unit manager's objective function, as shown below, does not have a solution.

Consequently, we abandoned the analytic approach of coding the boundary value problem in Matlab in favor of approximating a solution numerically with the Solver function of Microsoft Excel.

Unit Operator Objective Function: $$\max_{\{S(t)\}} \int_0^{\infty} (P(t)Q_{it}(1 - LR_{it} - ST_{it}ELF_i(Q_i(t))) - CCF(Q_i(t), S_i(t)))e^{-\rho t}dt$$

$$\text{s.t. } \frac{dS(t)}{dt} = -Q(t) \;:p(t) \qquad Q(t) \geq 0,\; S(t) \geq 0,\; S(0) = So$$

Production Cost Function: $$Cost_i = (c_1 Q_i^{c2} S_i^{c3})(1 + (c_7 + c_8 YrIL + c_9 YrIL^2 + \ldots + c_{13} YrIL^6 - 1)/Dmp)\,(W(Q,S)/((c_{14} + c_{15}Q + c_{16}S) * DRTS\_M)$$

Wellhead Value: $$price_t = P(t) = c_4 Year^2 + c_5 Year + c_6$$

First Order Conditions: $$p(t) = P(t) - MC = P(t)(1 - LR_i - ST_i ELF(Q_i(t))) - ST_i Q_{it} P(t)\frac{\partial ELF}{\partial Q} - \frac{\partial CCF}{\partial Q}$$

$$\frac{dp(t)}{dt} = \frac{\partial CCF(S(t),Q(t))}{\partial S} + \rho p(t) = \frac{\partial CCF}{\partial S} + \rho[P(t)(1 - LR_i - ST_i ELF(Q_i(t))) - ST_i Q_{it} P(t)\frac{\partial ELF}{\partial Q} - \frac{\partial CCF}{\partial Q}]$$

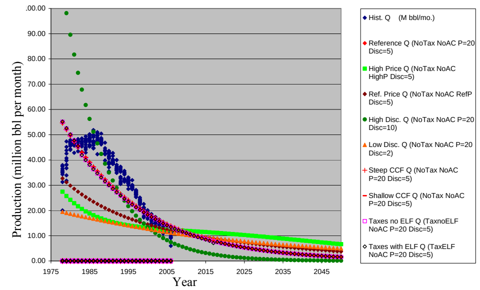
$$\lim_{t\to\infty} p(t)S(t)e^{-\rho t} = 0$$

Associated derivations become complex…

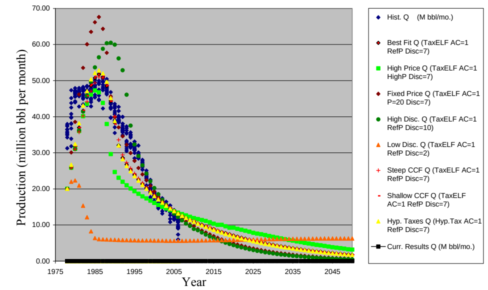
$$ELF(Q_i(t)) = (1 - \frac{300 * WELLS_{it} * DAYS_t}{Q_{it}})^{(\frac{150{,}000}{Q_{it}/DAYS_t})^{1.5333}}, \quad \frac{\partial ELF(Q_i(t))}{\partial Q} = [(vu^{v-1})(\frac{du}{dQ}) + (\ln u)(u^v)(\frac{dv}{dQ})][\frac{\partial}{\partial Q}g(f(Q))][\frac{\partial}{\partial Q}f(Q)], \quad \frac{d}{dt}(\frac{\partial ELF}{\partial Q}) = \{\frac{d}{dt}[(vu^{v-1})(\frac{du}{dQ})] + \frac{d}{dt}[(\ln u)(u^v)(\frac{dv}{dQ})]\}[\frac{\partial f(Q)}{\partial Q} * g'(f(Q))] + \{\frac{d}{dt}[\frac{\partial f(Q)}{\partial Q} * g'(f(Q))]\}[(vu^{v-1})(\frac{du}{dQ}) + (\ln u)(u^v)(\frac{dv}{dQ})]$$

Can A Model Approximate History?

Prudhoe Oil Production, Historical and Modeled



Prudhoe Oil Production, Historical and Modeled



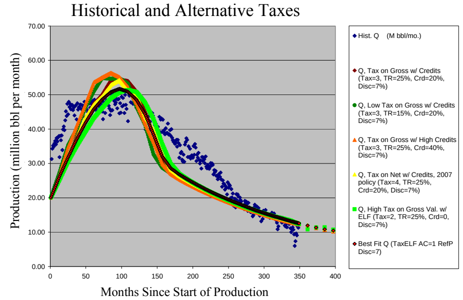
The plots on the left show discrepancy between model results (in a rainbow of colors) and actual production (blue dots) when modeling with no adjustment cost or constraint on initial production rate (i.e., a model of purely theoretical dynamic optimization).

The plots on the right show good model fit to actual production data when constrained to historical initial production and calibrated with discount rate (7%) and adjustment cost. The application of the calibrated model is simulating alternative tax policies (see below).

Alternatively, we can evaluate the difference between modeling and production history by using a "reasonable" discount rate and adjustment cost.

The Impact of Tax Policy on Production

Prudhoe Bay Oil Production, Historical and Modeled
Historical and Alternative Taxes



A "first-best" tax policy does not distort the dynamic optimization of oil production, thereby maximizing the total surplus (defined as producer profit plus tax revenue). In this case, a consistent tax rate on the gross value is first-best (the green path); alternative tax rates change the allocation of surplus between producer profit and tax revenue, but do not influence the optimum production path.

The "Best Fit" production path is for the historical severance tax policy, which was a tax on gross value; an increase in the tax rate in 1981 shifts the optimum production path to earlier periods because the model presumes perfect information of future tax (and price) conditions. Future modeling will consider the impact of imperfect information on the production path and net social benefit (i.e., unforeseen tax increases).

In 2007, the Alaska Legislature changed severance tax policy to levy the tax on net profits rather than gross revenue and to include a credit to offset investment costs. Modeling of these policies suggest they are effective in shifting production to earlier periods, but at the cost of lower total surplus.

CONCLUSIONS & FUTURE WORK

Dynamic modeling of oil production in Alaska proved too complex for an analytical solution, but tractable with a numeric model. Subject to uncertainty in the cost function estimation and calibration, the model can be used to simulate the impact of tax policies on optimum production paths, producer profits, and tax revenues, and can be used to evaluate the difference between economic theory and reality (although interpretation as sub-optimal production vs. modeling error remains elusive).

Future work will diverge on two paths. On one hand, we will seek greater realism (i.e., complexity) in modeling Alaska oil production by, for example, including a relationship between extraction rate and ultimate recovery and by making Trans-Alaska Pipeline System (TAPS) sizing endogenous to the modeling. On the other hand, we will also seek further understanding of the benefits and limits of dynamic modeling of energy production decisions by applying these methods to distinctly different energy resources, like renewable rather than finite resources (e.g., wind and biomass).

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